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Developing Number Sense in Pre-K with Five-Frames

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Abstract Teachers in early childhood and elementary classrooms (grades K-5) have been using ten-frames as an instructional tool to support students' mathematics skill development for many years. Use of the similar five-frame has been limited, however, despite its apparent potential as an instructional scaffold in the early elementary grades. Due to scant evidence of teacher use and a lack of systematic research we know little to nothing about both the developmental and pedagogical implications of using five frames and related instructional manipulatives in early childhood mathematics classrooms. In this paper, we provide an overview of five-frames and specifically demonstrate ways that five-frames, if used in conjunction with concrete manipulatives, can support pre-kindergarten (pre-K) children's development of Gelman and Gallistel's (1978) three basic counting principles: the stable-order principle, one-to-one correspondence, and cardinality. We conclude by discussing the developmental and instructional implications of using five-frames, as well as offer a set of teaching tips designed to help teachers maximize the potential advantages of integrating five-frames in the pre-K classroom.

Keywords Number Sense · Counting Principles · Pre-K · Mathematics · Five-Frames

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Introduction

The early childhood years, from age 0–8, serve as perhaps the most important developmental years of one's life. In recent years, the importance of this development has been recognized and embraced by teachers, parents, and researchers, particularly in the area of mathematics. In response to this growing understanding of the need for mathematical development, for the first time, the National Council of Teachers of Mathematics (NCTM 2000) identified *Principles and Standards for School Mathematics* that included a "pre-kindergarten" (pre-K) age band. Consequently, a major goal in pre-K classrooms has been to meet the guidelines proposed by NCTM. In this paper, we describe how pre-K teachers can leverage a simple instructional tool, the five-frame, to support children's development of key number sense skills outlined by NCTM.

Five-Frame Description

A five-frame (see Fig. 1) is simply a 1×5 row of squares that allows users to place physical manipulatives (dots, counters, coins, etc.), each within a single box, to create a visual representation for numbers zero–five. Similarly, traditional ten-frames (Fig. 2) are 2×5 arrays are

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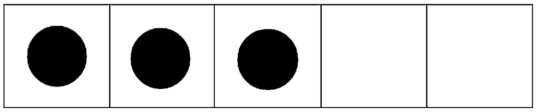

Fig. 1 Five-frame representation of the number three

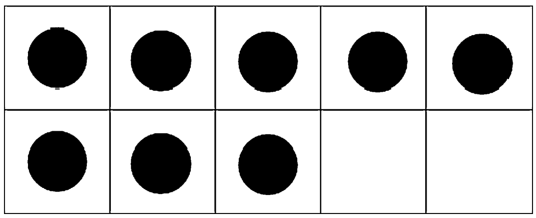

Fig. 2 Ten-frame representation of the number eight

designed to serve the same purpose as five-frames, but extend representations of numbers to zero through ten.

In this paper we consider two instructional variations of five-frames, describing their potential as both instructional scaffolds (e.g., assisting children in counting, partitioning, and tagging concrete objects) as well their capacity to serve as concrete analogs (Boulton-Lewis 1998) to represent numerical quantities and help students establish connections between different numerical representations.

We believe that five-frames will serve as a useful tool to support pre-K students' development of number sense concepts for several reasons. First, five-frames provide a basic and consistent organizational structure that inherently anchors mathematics around the number five, an important benchmark number in children's mathematical development (Novakowski 2007). Second, due to the limited number of squares in a five-frame, students are automatically constrained to working with smaller set sizes (five or fewer) that are well within their developmental counting range (Baroody 2009; Wynn 1990). This constraint reduces the cognitive load and task demands (Sweller 1994) and has also been shown to minimize related errors in counting studies (Frye et al. 1989; Gelman and Meck 1983). Third, five-frames present opportunities for children to establish connections between different numerical representations, a critical skill in one's mathematical development (NCTM 2008; NRC 2009). Fourth, five-frames allow children to explore combinations of numbers and observe part-to-whole relationships, an important consideration during early mathematics (Copley 2000; Fischer 1990; Hunting 2003). Finally, five-frames are visually and conceptually similar to ten-frame representations, and therefore early exposure to five-frames will help to familiarize students with an instructional tool commonly used in later elementary mathematics.

In addition to the developmental support that five-frames can provide to students, five-frames offer several distinct advantages for pre-K teachers. Five-frames are simple, very easy to make, and inexpensive. Classroom teachers can easily create their own five-frames using a basic computer program (e.g., Microsoft Word, Paint, or Photoshop) or construction paper so that each student can have their own. Furthermore, five-frames can be printed on cardstock or laminated so that they can be used in multiple lessons. The design or size of the boxes of the frame can be modified so that a variety of common physical manipulatives (e.g., snap cubes, teddy bear counters, and blocks) can be used in conjunction with a given five-frame activity. Finally, five-frames are not bound to any particulary early childhood mathematics curriculum, and therefore teachers can think creatively about ways to integrate five-frames into a variety of lessons, games, and daily activities intended to target the development of students' number sense.

Importance of Early Number Sense Development

Due to the increased calls for reform in early childhood mathematics that collectively advocate for more learning time specifically devoted to number and operation-related concepts than any other mathematical topic (e.g., NCTM 2008; NRC 2009), there has been a growing interest among curriculum developers and researchers in the instructional approaches that support children's conceptual development of number sense (NRC 2009; Sophian 2007; Van de Walle 2003). The rationale for this increased focus stems from a research base supporting both the need for number sense development (Jordan 2007) as well as the increasing realization that number sense is one of the most fundamentally important concepts to be developed in early mathematics (Baroody 2009; Jordan 2007; Kilpatrick et al. 2001; McGuire and Wiggins 2009; NCTM 2008; Van de Walle 2003). Research suggests that targeting number sense development in early mathematics prepares students to learn more complex mathematics concepts such as place value (Miura et al. 1993; Van de Walle 2003), part-to-whole number composition and decomposition (Fischer 1990; Hunting 2003), and basic arithmetic operations involving addition and subtraction (Levine et al. 1992; Wynn 1990). More generally, early exposure to key number sense concepts equips students with an understanding of core mathematical properties (NMAP 2008) and promotes numerical fluency (Baroody 2009). Conversely, inadequate development number sense in early elementary can be detrimental to one's long-term mathematical success if not fostered with early intervention (Jordan 2007; Jordan et al. 2007; Tzuriel et al. 1999; Wright et al. 1996). Early mathematical difficulties exacerbate children's risk for later school failure (Natriello et al. 1990).

Number sense skills developed in pre-K and kindergarten are not only foundational, but also correlated with first grades mathematics achievement (Jordan et al. 2006) with a specific correlation demonstrated between the mastery of counting principles and arithmetic abilities (Stock et al. 2009). Additional studies (Duncan et al. 2007; Ginsburg and Allardice 1984) have found that number sense skills developed in early elementary can be highly predictive of one's mathematics achievement as late as high school. Despite this growing body of empirical research and conventional wisdom surrounding the importance of developing number sense in pre-K, many classrooms do not capitalize on the wealth of available opportunities to formally develop students' informal number sense knowledge (Clements 2001).

Targeting Number Sense in Pre-K

While the specific definitions of number sense vary somewhat across mathematics researchers (Berch 2005), we have adapted Siegler's work (1991) in offering this abbreviated definition of number sense, one which we feel is developmentally appropriate for pre-K mathematics: Number sense is knowledge that can be demonstrated by identifying written numbers, performing counting activities, organizing numbers in sequence, and making decisions about magnitudes (i.e., comparisons between quantities). Because our primary goal of introducing five-frames in pre-K instruction is to equip students with the foundational skills for success in kindergarten mathematics, we call on the work of Jordan et al. (2006) to provide a more extensive framework of number sense components that should be targeted in kindergarten mathematics instruction.

Jordan et al. (2006) cite five key elements thought to govern the basic development of number sense in kindergarten children, all of which have been validated by research and are considered relevant to the math curriculum delivered in primary school (Griffin and Case 1997). These elements include: (1) Counting–grasping the one-to-one correspondence principle, stable-order principle, and understanding cardinality; (2) Number Knowledge–discriminating and coordinating quantities, and making numerical magnitude comparisons; (3) Number Transformation–transforming sets through addition and subtraction, calculating in verbal and nonverbal contexts, calculating with and without referents (physical and verbal); (4) Estimation–approximating or estimating set sizes, using reference points and; (5) Number Patterns–copying, extending or discerning numerical patterns and relationships.

Although it is theoretically possible to support all of these number sense components with five-frames, in our proposed framework, we specifically target one key elements: counting. In the sections that follow, we cite relevant developmental theory and make connections between ways that five-frames can directly support students' growth in these critical mathematical areas at a level that is developmentally appropriate for pre-K mathematics learning environments. We provide visual representations (where applicable) to display the key features of five-frames and model how they can promote numerical flexibility and support children in acquiring basic number sense skills.

The Principles of Counting

Gelman and Gallistel (1978) proposed that pre-K students' counting development is governed by five key counting principles, namely: (1) one-to-one correspondence–understanding that each object to be counted should be tagged with one and only one unique numeric tag; (2) the stable-order principle–knowing the number-name list (i.e., *one, two, three*…) must be used in a fixed order every time a group of objects is counted across all trials; (3) the cardinal principle–the number tag used for the last object in a count symbolizes the total number of objects in a set; (4) the abstraction principle–any types of objects can be counted together in a set; and (5) the order-irrelevance principle–the order in which objects are counted does not matter as long as none of the other counting principles are violated.

It has been suggested that the first three counting principles form the skeletal core for children's emerging counting knowledge because they alone indicate "how-to-count" (Gelman and Meck 1983). The final two principles, order-irrelevance and abstraction principle, have been categorized as non-essential features of counting (Stock et al. 2009), because violation of these principles does not result in incorrect counting (Briars and Siegler 1984). In other words, the order-irrelevance and abstraction principles are independent of the first three principles. Therefore, for the purposes of this paper we specifically explore how five-frames can support children's development and acquisition of the first three counting principles and related rules for understanding how-to-count.

Developing Counting Skills in Pre-K

Adults typically underestimate the complexity of our number system and take for granted the intricacies involved in acquiring even the most basic counting skills. Several researchers have documented the high level of skill required for counting (Bramald 2001; Resnick 1983) by describing the challenges children face in learning to coordinate the prerequisite counting skills (Ginsburg and Allardice 1984; Gelman and Meck 1983; Siegler 1991). Despite the non-trivial acquisition of counting skills, research suggests that the majority of students possess a baseline of counting related skills and procedures before entering pre-K (Clements 2004) and that many teachers and adults underestimate children's ability to engage cognitively demanding counting exercises (Clements 2009).

Specific research conducted on developmental trajectories of children's acquisition of Gelman and Gallistel's (1978) three essential counting principles has revealed that mastery of the three principles does not occur simultaneously (Stock et al. 2009), but rather that knowledge of the stable-order principle appears to develop first, followed by one-to-one correspondence, with a mastery of the cardinality principle developing the slowest (Butterworth 2004). In response to this developmental sequence, pre-K mathematics instruction tends to focus primarily on concrete counting skills (e.g., rote counting) because these exercises closely match children's earliest skill development and reflect their initial understanding of the stable-order principle. While the development of rote counting skills are critically important, a lack of exposure to more challenging counting exercises in pre-K may prevent children from developing an understanding of the principles that underlie counting (Siegler et al. 2006).

Therefore, in the next section, we outline theoretically informed methods supporting the use of five-frames to scaffold children's development of the three essential counting principles. We argue that this theoretical model of using five-frames to support number sense instruction can simultaneously accommodate children's developmental needs and move pre-K mathematics instruction beyond simple rote counting exercises to more abstract understandings of number.

One-to-One Correspondence Principle

Counting at a correct one-to-one correspondence is defined as one's ability to successfully label each object in a counting sequence with the correct number word (see Fig. 3). To successfully adhere to this principle, a child must coordinate two processes simultaneously, partitioning and tagging. In other words, a child must understand that each individual item to be counted needs to be transferred from the "to-be-counted" category to the "already-counted" category (partitioning) while a distinct numeral word is assigned, not to be used again in the counting sequence (tagging) (Marmasse et al. 2000). Figure 3 demonstrates a correct representation of the one-to-one counting principle for a set involving four items. The arrows indicate a correct understanding of the one-to-one correspondence principle; in other words, each counting word (*one, two, three, and four)* represents one and only one of the objects being tagged. Figure 4 demonstrates an example of an incorrect one-to-one correspondence, with the child erroneously double-counting the second object in the set, resulting in an incorrect cardinal value.

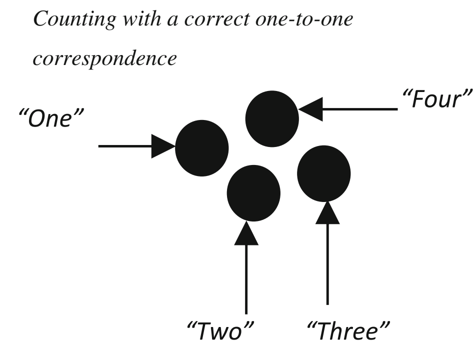

Fig. 3 Counting with a correct one-to-one correspondence

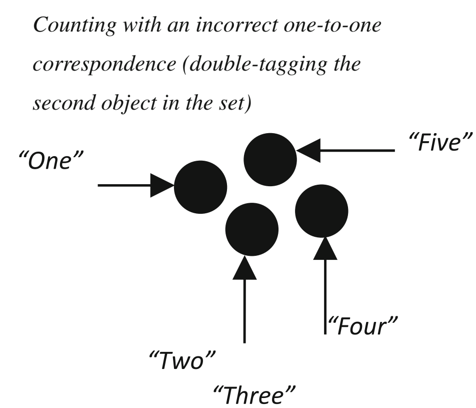

Fig. 4 Counting with an incorrect one-to-one correspondence (double-tagging the second object in the set)

Gelman and Meck (1983) suggested that children as young as 3 years old know in principle what it means to count at a one-to-one correspondence but that they may have difficulty applying this knowledge in practice. In a series of related studies, Gelman and Meck (1983) demonstrated that three- and four-year old children could accurately identify one-to-one counting errors (e.g., items skipped, double counted, starting count in the middle of array) made by a puppet. The results of Gelman's study also suggest that reducing the level and number of performance demands can improve children's ability to identify one-to-one counting errors. For this reason, we advocate for initial use of smaller sets of up to five objects (easily accommodated with five-frames), to help children develop a more thorough understanding of counting principles before moving on to larger numbers.

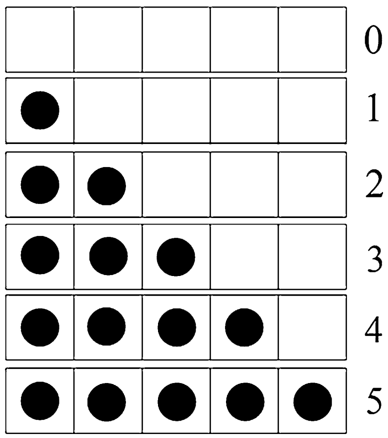

Fig. 5 Pre-constructed five-frames displaying the representations of numbers 0–5

In fact, pre-constructed five-frames (see Fig. 5) can serve as an instructional model to help reduce the cognitive and performance demands placed on children in basic counting tasks (Sweller 1994), while directly supporting children's understanding of the one-to-one correspondence principle. The pre-constructed five-frames essentially serve as a visual representation of quantity and inherently demonstrate the purpose of the frames as a unique tagging tool for each unitary item as well as establish the connection between quantity and the Arabic representation. Boulton-Lewis (1992) classify this type of visual representation as a *concrete analog*, suggesting its ability to "facilitate the construction, understanding and retrieval of mathematical concepts" (p. 220) and increase the flexibility in students' thinking by supporting transitions between concrete and abstract levels of understanding. If students begin by individually exploring the six different pre-constructed representations of five-frames (see Fig. 5), we propose that it will reduce the likelihood that students inaccurately tag an object with more than one unique numeric tag (violating the principle of one-to-one correspondence).

Using Manipulatives to Support One-to-One Correspondence

After children have had an opportunity to explore the six different pre-constructed five-frame representations and have been provided a basic overview of how the frames work, they can then begin using physical manipulatives and blank five-frames to practice counting at a one-to-one correspondence, thus transitioning to more abstract counting opportunities. One of the primary advantages of the frames is that they can assist students in systematically separating objects that have already been counted from those that still need to be counted. This is a challenging counting task for many pre-K children, particularly for those whom do not yet have well-developed partitioning strategies (Copley 2000) and if the objects to be counted are not organized in a systematic fashion (Marmasse et al. 2000).

To minimize the potential challenges encountered by developing counters, we offer several suggestions intended to facilitate students' success rates. First, we recommend that teachers introduce counting exercises using a uniform manipulative (i.e., dot counters, Teddy Bear Counters, unifix cubes, or buttons) that are the same in size, shape, and color. This uniformity allows children to focus exclusively on the counting exercise at hand, without the added distraction of size, shape, or color variations (Copley 2000). For the purposes of developing one-to-one correspondence, Van de Walle (2003) indicates that concrete manipulatives should be placed in five-frames from left-to-right, without skipping spaces, to reflect the same convention for written words and to help students develop a "standard" procedure for inserting objects into five-frames. When teachers model this strategy, they can help students understand that each object being entered into the frame is being tagged as one and only one distinct unit, therefore reinforcing children's conceptual understanding of one-to-one correspondence.

To further help children develop the concept of one-to-one correspondence, they should be encouraged to physically place, then point to and touch each object as they count it (see Fig. 6). Research suggests that this form of gesturing helps children to maintain correspondence and coordinate number words with counted objects at a greater rate of success (Alibali and DiRusso 1999). In other words, the pointing strategy encourages rhythmic physical motions that focuses the child's attention on individual items, aiding in segmentation and reducing working memory demands of counting (Sweller 1994) by externally representing the child's position in the count (Clements and Samara 2007). This allows students to cognitively map the spoken word to the actual count and help them understand that each object in the frame represents the value of one unit.

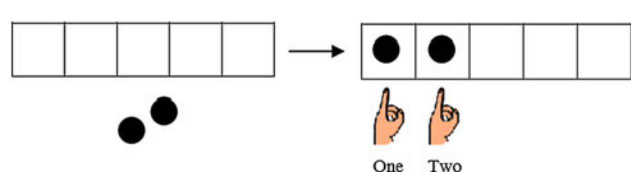

Fig. 6 Filling a blank five-frame with two uniform counters, with the proper pointing and modeling displayed

Stable-order Principle

Gelman and Gallistel's (1978) second counting principle, the stable-order principle, proposes that all counting words must be used in a fixed order every time a set of objects is counted. Although empty five-frames imply that a one-to-one correspondence exists between a physical manipulative and each square in the model, the frames themselves do nothing to suggest that the numeric counting tags chosen must be arranged in a stable (i.e., consistent) order. Despite the fact that pre-K children routinely practice verbally counting the correct sequence of number words "one, two, three, four…" many children encounter difficulty when attaching this count to physical objects. For example, a child developing counting skills may inaccurately tag a set of five objects, counting "one, two, four, five", thus skipping or forgetting to assign the tag of "three" to an object (Baroody and Wilkins 1999). Children who are learning to count may also be prone to pacing errors, in other words, saying the number names faster or slower than he/she is pointing. This particular counting error may partially stem from children's inability to coordinate multiple counting principles at once (i.e., stable-order principle and one-to-one correspondence).

We hypothesize that providing children who are learning to count with a five-frame containing the corresponding Arabic numeral representations (see Fig. 7) will support them in matching the numeric tag with the proper spoken counting word at the correct speed. Assuming that the student learns to begin filling the five-frame from left to right, without skipping any spaces in the frame, then he/she will essentially be forced to insert the manipulatives at the correct one-to-one correspondence. In other words, the frames act as "training wheels" for developing counters, assisting the students in setting up the proper one-to-one correspondence and tagging the numbers properly. Once the objects are inserted into the frame, students should practice pointing and counting with the proper tagging sequence. However, they will now have the corresponding numerical representations directly above the five-frame to support their counting.

For this instructional model to be successful, however, one very important assumption must be met. Students must be able to successfully recognize and identify the Arabic numerals 1-5 that are included in the top of the frame. Therefore, it may be helpful to provide students with extra practice in pointing and identifying the Arabic numerals before introducing manipulatives. This exercise is consistent with literature highlighting the importance of developing students' numeral recognition skills (Clements 2004) that prepare them for number processing and numerical reasoning tasks in later mathematics (Miura et al. 1993). In this case, the frames essentially work as a scaffolding mechanism and a visual representation, helping students to see that a one-to-one correspondence exists.

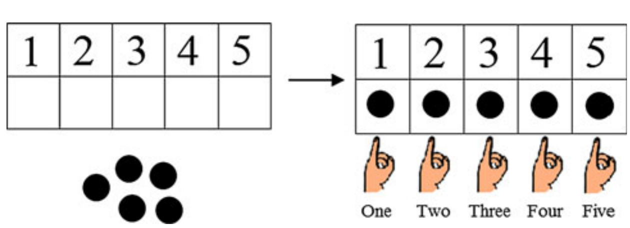

Fig. 7 Five-frame with corresponding Arabic numeral representations to aid in development of stable-order principle

Cardinality Principle

Developing an understanding of the cardinal value of a set (i.e., the ability to recognize the special significance that the last numeric tag of any count as representing the total number of objects) is arguably the most challenging of Gelman and Gallistel's (1978) *"how-to-count"* principle for students to acquire (Butterworth 2004; Fosnot and Dolk 2001). This difficulty could reflect, in part, that fact that cardinality is qualitatively different from the other how-to-count principles and appears to develop in a stage-like process (Bermejo et al. 2004) with children first learning the cardinal meanings within their limited counting range (e.g., number amounts 1, 2, or 3) (Wynn 1992). Producing the correct cardinal value of a set is further complicated because in order for students to arrive at the correct amount, they must properly coordinate the other two counting principles (one-to-one correspondence and stable-order) simultaneously and understand that cardinality is not only dependent on the counting, but also on the significance of the counting as it relates to quantity (Wynn 1990).

Despite the challenges and various levels of understanding associated with cardinality principle, there is no argument that cardinality is an important concept to be targeted in children between the ages of three and five (Clements and Samara 2007; Fuson 1988; Gelman et al. 1986; Wynn 1992) to help establish the primary goal of counting and also provide a foundation for more complex operational skills involving addition and subtraction (Fuson 1986). Clements and Samara (2007) go as far as suggesting that children's ability to connect the counting of objects in a collection to the number of objects (i.e., cardinality) in that collection serves as the "capstone of early numerical knowledge, and the necessary building block for all further work with number and operations" (p. 476). Therefore, it is critical to include opportunities that expose pre-K children cardinality concepts to help them begin to make connections between the words in the counting list and distinct numerical quantities they represent (Wynn 1990).

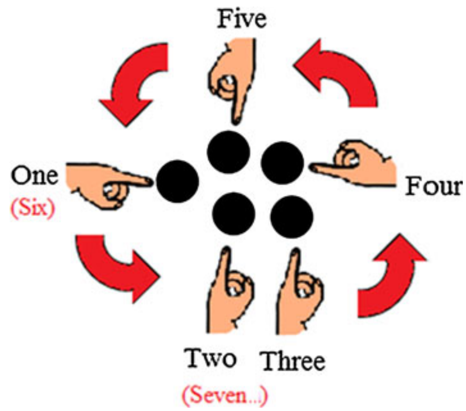

Fig. 8 Incorrect count of a set of five dot counters. The student continues to cycle through and tag individual objects more than once

Perhaps the most important advantage of using five-frames to reinforce the principle of cardinality is their potential to prevent incorrect counting habits. For example, a common mistake for many developing counters is to re-tag a set of items, particularly when the set of objects becomes larger. Most likely this occurs because students do not yet have the ability to mentally partition items that have been counted from items that still need to be counted (Copley 2000). As a consequence they double-tag single items or continue to count as high as possible (see Fig. 8), thus arriving at the incorrect cardinal value. Figure 9 displays a conceptual example of how five-frames can help support a developing counter, providing a conceptual "stop sign" for students. Again, for students to understand when to stop, this action should be consistently modeled by teachers with a variety of set sizes and different types of manipulatives.

In this case, using the five-frame in conjunction with the corresponding Arabic numerals above each space (as in Fig. 9) can effectively demonstrate all three of Gelman's "how-to-count" principles–one-to-one correspondence, stable-order principle, and cardinality principle–simultaneously. In addition to helping establish cardinality, five-frames used in conjunction with manipulatives and Arabic numeral representations help students to make connections between physical quantities, abstract digit symbols, and corresponding number words. At this level, however, combining multiple representations of number could be problematic, as students may be prone to confusing the notions of cardinality (total amount of objects in a set) with ordinality (the position or order of an object within a set). Therefore we encourage teachers to emphasize the Arabic numerals included above each dot in the five-frame are intended to represent the cardinal value of a given set (e.g., there are four squares filled in so that means we have four dots altogether), assuming that the five-frame is filled from left-to-right, leaving no squares blank.

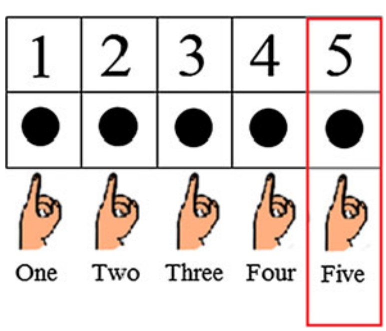

Fig. 9 Five-frame demonstrating a conceptual "stop sign" for students

Discussion

Despite the growing attention given to the development of children's number sense skills in pre-K mathematics there is still much more to learn. As research suggests, young children use mathematical ideas every day (Clements 2001) and develop informal skills that are surprisingly complex and sophisticated (Zur and Gelman 2004). Therefore the failure to capitalize on this development in preschool classrooms would represent a substantial missed opportunity in the critical years of mathematical development (Clements 2001). We are confident that five-frames can be leveraged as a simple, yet effective instructional tool to help bridge the gap between students' informal mathematics knowledge and their future mathematics learning, allowing them to become more proficient, flexible, and confident with basic counting tasks.

Recommendations for Best Practice

As previous research suggests (Uttal et al. 1997) many early elementary children encounter difficulty-making connections between concrete manipulative representations and more abstract mathematics (e.g., a student can solve 103 + 52 with base-ten blocks, but struggles to solve 14 + 12 with pencil and paper). Because five-frames and related manipulatives are intended to serve as concrete models and representations for pre-K students, teachers must be able to assist students in making the connections between the concrete and abstract meanings. In other words, even when working with very simple counting exercises or word problems, teachers must help students to realize that the five-frames and related manipulatives are symbols that have mathematical meaning.

A critical first step in maximizing the potential of five-frames involves providing teachers with professional development opportunities specifically designed to target their pedagogical content knowledge in mathematics. For example, improving teachers' ability to leverage five-frames to correct students' error patterns, identify developmental misconceptions, and facilitate basic problem solving tasks would be a great start. This type of targeted professional development will allow teachers to become more comfortable and confident in their instruction and improve their ability to effectively discuss with students the concrete and abstract mathematical meanings that five-frames convey. As a result, these teachers will be better prepared to accommodate their students' developmental needs and modify their instruction accordingly.

To summarize our suggestions for best practice, we have provided a table of teaching tips (see Table 1) based on a review of related literature, designed to help pre-K teachers properly integrate five-frames into their everyday instruction and maximize their potential effectiveness.

Recommendations for Future Research

Although our case for implementing five-frames in pre-K mathematics classrooms is based on an extensive review of developmental literature, research has yet to be conducted on the specific effects of five-frames in relation to child outcomes. Therefore we recommend empirically based studies to measure the effectiveness of five-frames in developing and supporting children's number sense skills.

Table 1 Teaching tips to facilitate pre-K students' number sense development with five-frames

Tip	Rationale	Literature support
Begin by using pre-constructed five-frames to orient students to a concrete, analog representation	Introduces students to five-frames with minimal cognitive demands (no manipulatives)	Sweller (1994)
	Models the purpose of five-frames to establish one-to-one correspondence	Boulton-Lewis (1998)
Count images or physical manipulatives in five-frames from left to right	Reflects the same convention used for written words and the number line	Van de Walle (2003)
	Provides a consistent expectation of how physical objects will be counted with five-frames	
Scaffold students' counting of concrete objects by encouraging them to physically point and touch each object to be counted	Helps children maintain one-to-one correspondence and coordinate number words with objects	Alibali and DiRusso (1999)
	Helps students develop rhythmic physical motions that focus on individual items to aid segmentation of counting by externally representing the child's position in the count	Clements and Samara (2007)
Use similar concrete manipulatives during counting exercises and problem solving activities with developing students	Lack of variation within the manipulatives allows children to focus on the counting without the distraction of size or color variations	Copley (2000)
Model the proper use of five-frames and related manipulatives directly	Helps students see the relevance and usefulness of five-frames	Sarama and Clements (2007)
	Establishes relationships between numbers and mathematics problem solving	Kelly (2006)
Consistently implement and integrate five-frames into a variety of instruction	Instructional variety provides students with different types of experiences and situates learning in multiple contexts (e.g., building, book readings, pretend, games) that promote number sense specifically anchored around the number five	Van de Walle (2007)
		Novakowski (2007)
Include prompts, hints or assistance to encourage students to solve basic problems involving five-frames and manipulatives	Simple questions such as, "*How do you know?*", "*Can you show me how you did that?*", and "*Why*" can be surprisingly motivating for young children. Also, this feedback helps to engage students in *thinking* and supports students' ability to connect concrete and abstract meanings	Sarama and Clements (2007)
		Ashlock (2006)
Do not assume that students automatically interpret manipulatives and corresponding five-frame representations in the manner that they are intended	Manipulatives and related representations can be inherently confusing to students and at times can interfere with learning. Ask students to describe what the manipulatives represent and correct any misconceptions or misinterpretations that they may have	Uttal et al. (1997)

Due to the ubiquitous use of concrete manipulatives in early childhood mathematics and the general assumption that they assist children in acquiring conceptual knowledge (McNeil and Uttal 2009; National Council of Teachers of Mathematics 2000; Sowell 1989; Van de Walle 2007), a logical extension seems to be one that explores the effects of using five-frames and manipulatives, perhaps in story-based or basic problem solving contexts. This will help to better identify how students do (or do not) understand the symbolic relationships between five-frames, manipulatives, and numbers. Additionally, we need a stronger literature base to effectively describe the best forms of guidance to support students' use and proper teacher facilitation of manipulatives, particularly in pre-K classrooms.

We believe that five-frames represent simple, yet powerful instructional tools that have the potential to make a significant impact in pre-K classrooms, with subsequent implications for later mathematics learning. Despite these potential advantages, we must resist the temptation to push difficult, more complex mathematics (e.g., multiplication and division; word problems where the change or start is unknown) into pre-K classrooms (Clements 2001). Instead, we must focus on the ways that five-frames can promote number sense development by helping children to interpret numerical relationships and develop reasoning strategies (Baroody 2009). We hope that pre-K teachers will be receptive to this instructional tool and begin to consider the ways that five-frames can supplement their own teaching practice and open up possibilities for student learning. If so, teachers can begin to incorporate frames into their daily instruction, finding ways to create fun and engaging activities that challenge students to think critically while developing stronger number sense skills.

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